

Available online at www.sciencedirect.com



International Journal of Heat and Mass Transfer 49 (2006) 836-843

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Transitional and turbulent heat transfer of swept cylinder attachment line in hypersonic flow

E. Benard *, R.K. Cooper, A. Sidorenko

School of Aeronautical Engineering, Queen's University Belfast, Belfast, Northern Ireland BT9 5AG, UK

Received 21 April 2005 Available online 2 December 2005

Abstract

A wide range of experimental results carried out in low enthalpy facilities is used to sustain simple correlations predicting heat transfer rates on an attachment line in hypersonic flow. It is observed that for long swept cylinders a nearly asymptotic state can be reached for which transition Reynolds number does not change substantially quite far from the apex of the cylinder. Intermittency based on measured heat fluxes was used to describe the laminar-turbulent process. Prediction of turbulent heat transfer rates is established using a reference temperature method that provides quick and relatively accurate estimates for a wide range of Mach number and temperature ratio. © 2005 Elsevier Ltd. All rights reserved.

Keywords: Correlation; Hypersonic flow; Transition; Turbulent; Swept cylinder

1. Introduction

As the space transportation industry is still looking for economical and highly reliable Earth-to-Orbit alternatives to the Space Shuttle, simple and relatively accurate prediction methods are useful for quick determination of aerothermal properties at a project stage. In reentry phase, the kinetic energy of the vehicle is dissipated by aerodynamic breaking, thus creating zones of intense heating. It has been demonstrated that knowledge of boundary layer status is essential in order to globally optimize the aerothermal design of spacecraft [1,2] as laminar to turbulent transition strongly increases heat transfer rates and requires larger provision of thermal protection, leading to a significant decrease of payload.

As all spacecraft use a swept wing configuration, laminar to turbulent transition exhibits a complex nature, but three main transition mechanisms are usually observed: the attachment line transition, the crossflow instability and the streamwise instability [3]. In this paper, focus is

0017-9310/\$ - see front matter @ 2005 Elsevier Ltd. All rights reserved. doi:10.1016/j.ijheatmasstransfer.2005.10.006

put on the first type as it preconditions the flow nature further downstream, as depicted in Fig. 1. Once the attachment line is turbulent, streamlines emanating from this location contaminate flow regions downstream and relaminarization is unlikely, as it would require high level of acceleration or massive suction. Along leading-edges and windward surfaces attachment-line transitional flows can develop [5,6] through the influence of freestream disturbances, or of any type of surface roughness originated from dust contamination, misalignment or destruction of the tiles of the thermal protection system.

During the development of high speed flight, attachment-line transition [7,8] and turbulence [9] have received considerable attention through wind tunnel tests on long swept cylinders. Those works were later synthesized and complemented experimentally by Poll [10]. Transition modelling efforts were started by studies of the linear instability [11] and were also extended to more general descriptions [12] and to non-linear stages [13].

Although a substantial body of experimental data has been gathered over the past decades, modelling of transitional and turbulent attachment line compressible flows is highly empirical [14] and is loosely connected to advances in instability theory and compressible turbulence models

^{*} Corresponding author. Tel.: +44 2890274180; fax: +44 2890382701. *E-mail address:* e.benard@qub.ac.uk (E. Benard).

Nomenclature

- D cylinder diameter (m)
- M Mach number
- *r* recovery factor
- \overline{R} $V_{\rm e}\eta/v_{\rm e}$, characteristic Reynolds number
- \overline{R}^* $V_e \eta^* / v^*$, Poll's Reynolds number computed with Poll's reference temperature
- St $\Phi_w/(\rho_{e0}V_eC_p(T_r T_w))$, Stanton number
- T absolute temperature (K)
- $T_{\rm r} = T_{\rm e}(1 + r\frac{\gamma 1}{2}\dot{M}_{\rm e}^2)$, recovery temperature (K)
- T^* $T_e + 0.1(\bar{T}_w \bar{T}_e) + 0.6(\bar{T}_r \bar{T}_e)$, Poll's reference temperature (K)
- *X*, *Y*, *Z* coordinate system, respectively chordwise, spanwise and normal to the cylinder surface
- U, V, W velocity components outside the boundary layer

Greek symbols

 η $v_{\rm e}/(dU_{\rm e}/dX)^{1/2}$, boundary layer length scale based on chordwise gradient of velocity (m)

[15–17]. This situation originates from the very small number of works detailing the structure of an unstable and transitional attachment line [4,18]. In most of the experiments carried out at high speed thermal methods of transition detection have been used, such as change of the adiabatic wall temperature in supersonic regime [19] and increase of heat transfer. Both detection methods have demonstrated that when transition is forced by a roughness element, a critical roughness height, below which no influence is detected, can be defined as a function of the local Mach number [20]. Within this context, it is also shown that the Reynolds number used in the studies of attachment-line flows following the definition proposed by Poll [14], and noted \overline{R}^* , is a valid parameter to predict transition for the case of large disturbances. When the attachment line surface is free of significant defects transition occurs at much larger Reynolds number, but the threshold is at



Fig. 1. Attachment line on a swept leading edge [4].

- $\begin{array}{ll}
 \rho & \text{density } (\text{kg m}^{-3}) \\
 \nu & \text{kinematic viscosity } (\text{m}^2 \text{ s}^{-1}) \\
 \Lambda & \text{sweep angle } (^{\circ}) \\
 \end{array}$
- $\Phi_{\rm w}$ wall heat flux (W m⁻²)

Subscripts

- e boundary layer edge conditions along the attachment line0 stagnation conditions
- tr transition conditions
- w wall conditions
- 1 onset of transition
- 2 end of transition
- ∞ free-stream conditions
- L laminar
- T turbulent

least a function of the local Mach number and of the wall to stagnation temperature ratio [21].

Following initial attempts of predicting turbulent attachment line heat transfer [22,23] the most significant contributions have been the ones proposed by Poll [14,24] in which correlations for transitional and turbulent regimes are built up as extension of low speed results. However, many authors have shown that Poll's approach, although simple to implement, does not correctly predict heat transfer rates in hypersonic flow [25-28]. A more refined approach has been developed by Bellone [25] on the basis of numerical simulations but introducing supplementary parameters in the more elaborate reference temperature model. Bellone showed that, based on a similar experimental database as used in the present paper, the standard deviation of the prediction against the experiments was 7%. The object of this paper is to discuss in more detail those aspects and to revisit Poll's ideas [14,24]. Experimental results for heat transfer in low-enthalpy environments are reported in order to build up a reliable database of attachment line heat transfer rates, along with other sources of experimental data. Using intermittency distribution for the analysis leads to a reasonable degree of confidence in the prediction of heat transfer, especially if a quasi-asymptotic state exists and if the transition threshold corresponds to the one obtained for bypass transitions. A first estimation of turbulent heat transfer is also proposed, which applies for a wide range of flow conditions.

2. Experimental methods

As a large part of experimental data exploited in the following paragraphs were obtained in the same facility

[26-28] important experimental parameters and procedures will be briefly stated. The 210 mm diameter blowdown wind tunnel of CEAT of Poitiers gives a freestream Mach number of 7.14 ($\pm 1.3\%$), at freestream unit Reynolds numbers between $7.5 \times 10^6 \leq Re_{\infty}/m \leq$ 35×10^6 , which were obtained by changing the stagnation pressure and temperature within the following range: $2.5 \le p_{0,\infty} \le 9.5 \text{ MPa}, \pm 0.5\% \text{ and } 650 \le T_0 \le 800 \text{ K},$ ± 800 K, $\pm 0.8\%$, respectively [28]. Circular cylinders with upstream ends cut off parallel to the freestream direction were mounted at various sweep angles thus allowing the Mach number at the edge of the boundary along the attachment line to vary, $3.28 \leq M_e \leq 6.15$. The accuracies for the cylinder radius and the sweep were ± 0.02 mm and $\pm 0.1^{\circ}$, respectively. This results in typical errors on Reynolds numbers of 1-2%, using a modified Newtonian model for the calculation of the chordwise velocity gradient.

Although this paper exploits an experimental database obtained in various facilities and instrumentations, most of the data were extracted on the basis of transient methods due to the relatively short test duration. The experimental results presented hereafter were deduced from the thin skin analysis which requires the measurement of temperature change of a thin element of material. All models were equipped, along the attachment line, with spot-welded Chromel-304L stainless steel thermocouples on the inner surface of a 304L grade stainless steel thin skin, of 0.5 ± 0.001 mm thickness. Such moderate thickness ensures that lateral conduction losses are small, less than 1% [28]. Heat capacity of the 304L grade stainless steel was known with accuracy of $\pm 3\%$. Some models were also pre-cooled before the tests by the use of gaseous nitrogen and maintained at low temperature during the test by circulation of liquid nitrogen in the model.

After starting the nozzle and injecting the model into the test section, the heat flux was deduced from the time evolution of the wall temperature. With limited contamination from the wind tunnel side wall during the injection, the uncertainty on the wall heat transfer rates was estimated at approximately 5% and errors on Stanton number are between 6% and 8% [28]. It should also be noted that the variation of the unit Reynolds number induces a change of the freestream Mach number, due to the variation of the thickness of the nozzle wall boundary layer. Typically a 2% variation of freestream Mach number within the unit Reynolds number range, inducing an increase of heat flux of about 9% with respect to nominal conditions.

By varying the tunnel stagnation conditions it was possible to determine the evolution of Stanton number with respect to the Reynolds number. The Reynolds number used in this study follows the definition proposed by Poll [14] and is denoted \overline{R}^* . Transition is indirectly detected by an increase of the Stanton number, and a transition Reynolds number corresponding to a minimum of the Stanton number can be defined with minimum ambiguity and is denoted \overline{R}^*_{trl} (Fig. 2).



Fig. 2. Typical transition case ($\Lambda = 60^{\circ}$, $M_{\infty} = 7.14$, $\frac{T_w}{T_0} \approx 0.38$, 0.30 mm diameter trip at Y/D = 10).

Physically, the boundary layer grows from the apex along the attachment line of the model and the threedimensional shock wave, attached to the apex, wraps around the model. Schlieren visualizations show that the stand-off distance of the shock in front of the cylinder reaches a constant value within typically 5–10 diameters. This represents the so-called infinite swept cylinder situation. On the other hand the measured heat transfer rates, in a given flow regime, also tend towards similar asymptotic values. Achievement of the asymptotic state for the viscous flow can be judged by comparison of the measured Stanton number values along the attachment line with the exact laminar solution, but can be masked by systematic errors attached to each thermocouple location (Fig. 2). When roughness elements were used to trigger transition they were put at large values of y/D, corresponding approximately to the start of the asymptotic flow region. The shock wave emanating from the roughness element was shown numerically [25] not to modify the heat transfer once the flow was fully turbulent.

3. Transitional regime modeling

For the sake of simplification, no distinction between "natural" and "bypass" transitions [29], small and large disturbance scenario, respectively, will here be considered because of the relative lack of detailed knowledge of transitional attachment-line boundary layers in supersonic and hypersonic regimes.

Dependence of Stanton number on local parameters, the characteristic Reynolds number, the boundary layer edge Mach number, and the relative wall temperature is examined. The final aim is to address the possible link between the extent of the transitional process (in terms of Reynolds number range) and general parameters, such as the transition onset Reynolds number, in a same way as for more generic flows [16].

The present analysis is an extension of Poll's work [14] which is an application of the formalism developed for the first time by Emmons [30]. Those works describe how to relate turbulent spot geometry to an intermittency

function, γ , which is defined as the fraction of observation time for which the flow can be considered turbulent, by means of the observation of a physical parameter. In a hypersonic context it is usual to choose the wall heat transfer rate, since this parameter is the most important for thermal protection design. Flow intervals between events are assumed to behave as pure laminar flow. This statement relies on the assumption that the rise and fall of heat transfer rate with time represent very short events in comparison to laminar and turbulent periods.

For given flow conditions, and at a particular location along the attachment-line, this means that the wall heat transfer rate can be expressed as a linear combination of laminar and turbulent heat transfer rates obtained at the same conditions:

$$St_{\rm e} = (1 - \gamma)St_{\rm e,L} + \gamma St_{\rm e,T} \tag{1}$$

Therefore determination of the intermittency requires the values of the Stanton number in laminar and in turbulent regimes, $St_{e,L}$ and $St_{e,T}$, respectively. Obviously defining the intermittency in such a way is rather indirect because it relies on a global property rather than intermittency inferred from measurements of the unsteady wall heat transfer. However, this approach offers the convenience of accommodating the large body of experimental data, which were mostly obtained with conventional techniques.

For a given configuration, in laminar regime, a reference Reynolds number \overline{R}_{\min}^* is taken at the lowest stagnation pressure. To offset any error on the local measurement of the heat transfer rate and using laminar similarity rule, it is possible to write:

$$St_{e,L}\overline{R}^* = St_{e,L,min}\overline{R}^*_{min}$$
 (2)

In that rule, $St_{e,L,min}$ is the measured value of Stanton number by a given sensor in a laminar regime for the Reynolds number \overline{R}_{min}^* .

In a turbulent regime, this procedure is more delicate to put in place because there is no formal similarity rule. However using the turbulent correlation of Poll [14], an empirical similarity rule can be expressed:

$$St_{e,T}\overline{R}^{*0.42} \approx St_{e,T,max}\overline{R}_{max}^{*0.42}$$
 (3)

In that formula, $St_{e,T,max}$ the Stanton number measured by the same thermocouple as previously but now in a turbulent regime for a reference Reynolds number \overline{R}_{max}^* , which corresponds to the highest stagnation pressure tested.

In some cases, the reference laminar or turbulent situations do not occur within the range of unit Reynolds number available in that wind tunnel and the reference values are to be evaluated through numerical or empirical methods. The laminar heat transfer rates can be computed through similar solution calculations [31] and the turbulent ones are deduced from empirical rules such as the one presented in the next paragraph.

Poll proposed the following intermittency distribution at a given location along an attachment-line [14]:

$$\gamma = 1 - \exp\left(-0.412\left(\frac{\overline{R}^* - \overline{R}_{trl}^*}{\lambda}\right)^2\right) \tag{4}$$

with $\lambda = \overline{R}_{\gamma=0.75}^* - \overline{R}_{\gamma=0.25}^*$, which represents the extent of the transitional regime in the Reynolds number space. \overline{R}_{trl}^* is the transition onset Reynolds number. The validity of that type of distribution was partially confirmed at high speeds, for more generic types of flow [32].

The calculation procedure is twofold, firstly computation of the intermittency factor of a run through (1)–(3) for each thermocouple, and secondly, after a complete sweep of the Reynolds number range, plotting the following function:

$$F(\gamma) = \left(\frac{-1}{0.412}\ln(1-\gamma)\right)^{1/2} = \frac{\overline{R}^* - \overline{R}^*_{\text{tr1}}}{\lambda}$$
(5)

For each location along the attachment line, a linear fit over the transitional range is estimated, then one extracts \overline{R}_{tr1}^* from the intercept and $1/\lambda$ from the slope (Fig. 3).

Even if the dispersion is appreciable for $F(\gamma) \leq 0.5$, or $\gamma \leq 10\%$, the extraction of the leading parameters is straightforward: for the above case $\overline{R}^*_{trl} \approx 227$ and $\lambda = 15$ for that particular location. Those values result in a good fit to the intermittency distribution as illustrated in Fig. 4.

Through the collection of pairs $(\overline{R}_{tr1}^*, \lambda)$ for all the locations along the attachment-line, it appears that the threshold of transition onset does not vary appreciably downstream of the source of disturbance [20]. However, the transitional extent, λ , changes quickly downstream of the roughness element and tends to reach an quasi-asymptotic value of $\lambda \approx 10$ for this particular case (Fig. 5). As both the threshold and the transitional extent values seem to be frozen in the spanwise direction, it implies that a mechanism of quasi-perpetual sustainability of the transition might exist. Other experiments [28] suggest that the intrinsic turbulent spot dynamics could result from a balance between spanwise growth and chordwise stretch of individual spots.



Fig. 3. Typical linear fit to the function F at Y/D = 13.55 ($A = 60^{\circ}$, $M_{\infty} = 7.14$, $\frac{T_{w}}{T_{w}} \approx 0.39$, 0.07 mm thickness trip at Y/D = 8.5).



Fig. 4. Typical fit to intermittency distribution at Y/D = 13.5 (same case in Fig. 3).



Fig. 5. Spanwise evolution of transitional extent (longer cylinder) ($M_e = 5.15$, $T_w = 300$ K, $T_0 = 750$ K trip at Y/D = 6.7, Ref. [26]).

A more singular behavior can be observed in some cases especially when the transition is "natural" (Fig. 6). In the plot of the *F* function, two slopes appear clearly with the second ramp established when $\gamma \ge 10\%$. This phenomenon is similar to the "subtransition" described by Narasimha [33] and it might be linked to the spot dynamics along a slender body, like the cylinders used in those experiments. But, those cases essentially occur when the local Mach number is high, $M_e \ge 6$, and for natural transition configurations. Therefore, specific effects of compressibility might affect the breakdown of boundary layer instability, and in turn the spot dynamics.

Another aspect of this analysis is to collect data on available transitional cases in order to look for a general link between the transition threshold and the transitional extent. Three sources dealing with hypersonic conditions have been used [26–28] and two subsonic cases [35,14] have been added as comparison points (Fig. 7).



Fig. 6. Double slope of the F function $(M_e = 6.06, T_w = 308 \text{ K}, T_0 = 733 \text{ K}, Y/D = 9.2, \text{ Ref. [27]}).$



Fig. 7. Transitional extent versus transition threshold.

There is no clear dependency of the pairs $(\overline{R}_{trl}^*, \lambda)$ to the local Mach number value, but it appears, although based on a small number of points, that there is a minimal transitional extent which could be crudely expressed as

$$\lambda_{\min} \approx 0.045 \overline{R}_{tr1}^* \tag{6}$$

This would suggest that transition cannot happen suddenly, even for quasi-asymptotic conditions. For $\overline{R}_{trl}^* \approx$ 250, which is the transition threshold for bypass configurations proposed in [19] by using Poll's approximation and since checked numerous times [20], a good approximation of the transitional extent could be deduced:

$$\lambda_{\overline{R}_{t-1}^* \approx 250} \approx 15 \pm 5 \tag{7}$$

4. Turbulent regime modeling

As an engineering tool, a simple but accurate estimation of turbulent heat transfer can be useful. As shown by many authors [25–28] Poll's formula [14], based on the reference

temperature T^* , does not correctly predict heat transfer rates in hypersonic flow. In order to build a more reliable tool, an extensive review of available resources has been carried out [22-28,34,36-49]. Only cases with Reynolds numbers clearly higher than the one corresponding to the transition completion have been selected in order to minimize the low Reynolds number effects on turbulence [50]. The selection incorporates a majority of data obtained far from the cylinder apex, that is as close as possible to quasi-asymptotic region. Only the methods with reasonable precision on wall temperature and heat transfer have been selected. For example, most of the relatively old data obtained by means of thermosensitive paints were rejected because of the lack of precise information on wall temperature and as paints are relatively intrusive. It has to be mentioned that many points [26–28] are the average of very close runs. In total 272 points have been collected, covering the range: $0.14 \leq T_w/T_0 \leq 0.96$ and $0.35 \leq M_e \leq 6.89$.

The numerical procedure is essentially the same as the one used by Poll, based on an extension of the reference temperature concept of Eckert [51]. The basis of the reference temperature method is to assume that the nature of the analogy between friction and heat transfer is not fundamentally modified by compressibility effects, assuming that turbulence production, diffusion and dissipation are only weakly affected by the external Mach number. A likely limitation to this simplistic analogy lies in complex links between "modes of turbulence" [52], i.e. rotational, entropic and pressure fluctuations, at supersonic speeds. In this review, Gaviglio [53] concluded that there is not yet a clear understanding and a relevant model for a generalised analogy between momentum and heat transfer through a turbulent boundary layer. The remark is also likely to be valid for the transitional case. Those difficulties reinforce the need of developing simple correlations.

In the case of the swept attachment line Poll used the following reference temperature:

$$T^* = T_e + K_1(T_w - T_e) + K_2(T_r - T_e)$$
(8)

with T_e , T_w , T_r being the boundary layer edge temperature, the wall temperature and the recovery temperature (with r = 0.89), respectively, and $K_1 = 0.1$, $K_2 = 0.6$. Poll's procedure is similar to Eckert's with the exception that the reference temperature also modifies the reference scale, based on the viscous scale, η^* , which appears into the Reynolds number definition, \overline{R}^* . This explain why the coefficients K_1 and K_2 are significantly different to ones initially proposed by Eckert for flat-plate flows. Poll took a value of Reynolds analogy, *s*, equal to:

$$s = \frac{St_e}{\left(\frac{Cf_e}{2}\right)} = 1.24\tag{9}$$

A different value of Reynolds analogy factor has been chosen which could be more compatible with current knowledge of turbulent compressible boundary layers [50,54,55]:

$$s = 1 \tag{10}$$

This choice of Reynolds analogy factor is reasonable as it is applied to a flow over a smooth surface, even the transition was itself potentially triggered by an isolated roughness located upstream the measurement position.

If, following Poll's strategy [14], the incompressible skin friction empirical law is used, it is proposed that heat transfer rates are computed using the following equation:

$$St_{e,T} = \frac{0.0689}{\overline{R}^{0.42}}$$
 (11)

The new Reynolds number, \overline{R} , is computed by means of a new reference temperature \overline{T} .

It has been concluded that the following choice of reference temperature produces the best fit to the data:

$$\overline{T} = T_{\rm e} + 0.52(T_{\rm w} - T_{\rm e}) \tag{12}$$

On average the standard deviation for the new correlation is 11.7% against 26.7% using Poll's reference temperature. However, it is recognised that the initial contribution from Poll was based a much smaller sample of experimental data, and therefore did not benefit from the more extensive and partially more accurate database. Only 5 points are over a deviation of 30% and 88 points are within 5% (Fig. 8). Dashed lines indicate $\pm 10\%$ deviation between the correlation proposed and experimental results and those boundaries approximately represent the experimental error attached to the measurement techniques exploited in the body of works. Overall the proposed correlation reproduces correctly the trend of dependency on wall temperature and Mach number over a wide range of those parameters. In particular, it can be deduced from Eq. (12) that the reference temperature does not appear to be Mach number dependent but rather dominated by the effect of wall temperature.



Fig. 8. New correlation for turbulent heat transfer versus experimental data.

5. Conclusion

It has been demonstrated that intermittency distribution is a reliable tool for the analysis of transitional boundary layers over swept cylinders in hypersonic flows. A reasonable degree of confidence in the prediction of heat transfer can be reached, especially if a quasi-asymptotic state exists and if the transition threshold corresponds to the one obtained for bypass transitions.

A new formula for estimation of turbulent heat transfer is proposed and applies for a wide range of flow conditions. However, there is still a need for experimental data for both laminar and turbulent regimes in order to improve the modelling strategies and to create the basis for more elaborated methods.

Acknowledgement

The first author is grateful to T. Alziary de Roquefort, CEAT Poitiers—France, for his constant advice and encouragement.

References

- [1] M.S. Holden, A review of aerothermal problems associated with hypersonic flight, AIAA Paper 86-0267, 1986.
- [2] E.V. Zoby, R.A. Thompson, K.E. Wurster, Aeroheating design issues for reusable launch vehicles—A perspective, AIAA Paper 2004–2535, 2004.
- [3] H.L. Reed, W.S. Saric, D. Arnal, Linear stability theory applied to boundary layers, Ann. Rev. Fluid Mech. 28 (1996) 389–428.
- [4] A.I. Semisynov, A.V. Fedorov, V.E. Novikov, N.V. Semionov, A.D. Kosinov, Stability and transition on a swept cylinder in a supersonic flow, J. Appl. Mech. Tech. Phys. 44 (2) (2003) 212–220.
- [5] D.I.A. Poll, Boundary layer transition on the windward face of Space Shuttle during re-entry, AIAA Paper 85-0899, 1985.
- [6] K. Fujii, Y. Inoue, Aerodynamics heating measurement on after body of hypersonic flight experiment, J. Spacecraft Rockets 35 (6) (1998) 736–741.
- [7] W. Pfenninger, Flow phenomena at the leading edge of swept wings, in: Recent Developments in Boundary Layer Research, AGARDograph 97(4) (1965).
- [8] M. Gaster, On the flow along swept leading edges, Aeronaut. Quart. XVIII (2) (1967) 165–184.
- [9] N.A. Cumpsty, M.R. Head, The calculation of three-dimensional turbulent boundary layers. Part II: Attachment line flow on an infinite swept wing, Aeronaut. Quart. XVIII (1967) 150–164.
- [10] D.I.A. Poll, Transition in the infinite swept attachment line boundary layer, Aeronaut. Quart. XXX (1979) 607–629.
- [11] P. Hall, M.R. Malik, D.I.A. Poll, On the stability of an infinite swept attachment line boundary layer, Proc. Roy. Soc. London, Ser. A 395 (1984) 229–245.
- [12] R.S. Lin, M.R. Malik, On the stability of attachment line boundary layers. Part 1. The incompressible swept Hiemenz flow, J. Fluid Mech. 311 (1997) 239–255.
- [13] P.R. Spalart, Direct numerical study of leading edge contamination, in: AGARD CP438: Fluid dynamics of three dimensional turbulent shear flows and transition, 5 (1988) 1–13.
- [14] D.I.A. Poll, The development of intermittent turbulence on a swept attachment line including the effects of compressibility, Aeronaut. Quart. XXXIV (1983) 1–23.
- [15] B.A. Singer, Modelling the transition region, NASA report, CR-4492 (1993).

- [16] D. Arnal, Laminar-turbulent transition, in: T.K.S. Murthy (Ed.), Computational Methods in Hypersonic Aerodynamics, Kluwer Academic Publishers, 1991, pp. 233–264.
- [17] D.A. Dilley, Evaluation of CFD turbulent heating prediction techniques and comparison with hypersonic experimental data, NASA report CR-2001-21-837, 2001.
- [18] C.P. Coleman, D.I.A. Poll, J.A. Laub, S.W.D. Wolf, Leading edge transition on a 76° swept cylinder at Mach number 1.6, AIAA Paper 96-2082 (1996) 1–12.
- [19] T.R. Creel, I.E. Beckwith, F.J. Chen, Transition on swept leading edges at Mach 3.5, J. Aircraft 24 (10) (1987) 710–717.
- [20] E. Benard, L. Gaillard, T. Alziary de Roquefort, Influence of roughness on attachment line boundary layer transition in hypersonic flow, Exp. Fluids 22 (1997) 286–291.
- [21] L. Gaillard, E. Benard, T. Alziary de Roquefort, Smooth leading edge transition in hypersonic flow, Exp. Fluids 26 (1999) 169–176.
- [22] I.E. Beckwith, J.J. Gallagher, Local heat transfer and recovery temperatures on a yawed cylinder at a Mach number of 4.15 and high Reynolds number, NACA Memorandum, 2-27-59L, 1959.
- [23] H.W. Coleman, B.C. Lemmon, Prediction of turbulent heat transfer and pressure on swept leading edges, J. Spacecraft 11 (6) (1974) 376–381.
- [24] D.I.A. Poll, Skin friction and heat transfer at an infinite swept attachment line, Aeronaut. Quart. XXXII (1981) 299–318.
- [25] F. Bellone, New models for the prediction of attachment-lines behaviour at hypersonic speeds, Ph.D. Thesis, Cranfield (2002), no. 51–14447.
- [26] J.L. Da Costa, Contribution a l'étude de la transition de bord d'attaque par contamination en ecoulement hypersonique, PhD thesis, University Poitiers, France, 313, 1990.
- [27] L. Gaillard, Etude de la transition de bord d'attaque sur cylindre en fleche en ecoulement hypersonique, PhD thesis, University Poitiers, France, 641, 1993.
- [28] E. Benard, Contribution a l'etude de la transition de couche limite sur un cylindre en fleche en ecoulement hypersonique, PhD thesis, University Poitiers, France, 1998.
- [29] M. Morkovin, On the many faces of transition, in: C.S. Wells (Ed.), Viscous Drag Reduction, Plenum Press, 1969, pp. 1–31.
- [30] H.W. Emmons, The laminar turbulent transition in a boundary layer, J. Aerospace Sci. 7 (1951) 490–498.
- [31] I.E. Beckwith, Similar solutions for the compressible boundary layer on a yawed cylinder with transpiration cooling, NACA report, TN4345, 1958.
- [32] F.K. Owen, C.C. Horstman, Hypersonic transitional boundary layers, AIAA J. 10 (6) (1972) 769–775.
- [33] R. Narasimha, On the distribution of intermittency in the transition region of a boundary layer, Progress in Aeronautical Sciences, 22, Pergamon Press, 1985, pp. 29–80.
- [34] T. Alziary de Roquefort, Progress in low enthalpy hypersonic wind tunnel testing, in: ESTEC/ESA Second European Symposium for Aerothermodynamics for Space Vehicles, 1994, pp. 575–582.
- [35] D. Arnal, J.C. Juillien, Etude de la transition et de la contamination de bord d'attaque sur ailes en fleche, in: AGARD Conference Proceedings, Fluid Dynamics of Three Dimensional Turbulent Shear Flows and Transition, CP 438, 1988, vol. 6, pp. 1–14.
- [36] M.S. Holden, J.M. Kolly, Attachment-line transition studies on swept cylindrical leading edge at Mach numbers from 10 to 12, AIAA Paper 95-2279, 1995.
- [37] E.A. Brun, G.B. Diep, B. Le Fur, Transport de chaleur et de masse sur des cylindres en fleche dans un ecoulement Supersonique, AGARDograph 97 (1965) 715–753.
- [38] G.B. Diep, Etude aerodynamique et aerothermique de cylindres circulaires en attaque oblique dans un ecoulement supersonique, Publications Scientifiques et Techniques, 432, Ministere de l'Air, 1967.
- [39] P.B. Burbank, R.A. Newlander, I.K. Collins, Heat transfer and pressure measurements on a flat plate surface and heat transfer measurements on attached protuberances in a supersonic turbulent boundary layer at Mach number of 2.65, 3.51, 4.44, NASA report, TND-1372, 1962.

- [40] D.M. Bushnell, Interference heating a swept cylinder in region of intersection with a wedge at Mach number 8, NASA report, TN D-3094, 1965.
- [41] D.M. Bushnell, Effect of shock impingement and other factors on leading edge heat transfer, NASA report, TN D-4543, 1967.
- [42] D.M. Bushnell, J.K. Huffinann, Investigation of heat transfer to a leading edge of 76° swept fin with and without chordwise slots and correlations of swept leading edge transition data for Mach number 2 to 8, NASA report, TM X-1475, 1967.
- [43] J.L. Hunt, D.M. Bushnell, I.E. Beckwith, The compressible turbulent boundary layer on a blunt swept slab with and without leading edge blowing, NASA report, TN D-6203, 1971.
- [44] R.A. Jones, R.L. Trimpi, Heat transfer and pressure distributions at a Mach number of 6 for 70° swept slab wings with sharp and spherical noses and cylindrical leading edges, NASA report, TM X-682, 1962.
- [45] R.A. Jones, Heat transfer and pressure investigation of a fin plate interference model at a Mach number of 6, NASA report, TN D-2028, 1964.
- [46] D. Arnal, Recherches sur la transition liees au projet Hermes. Rapport de synthese, ONERA-CERT (Toulouse), 18/5005.08 AYD, 1988.
- [47] D. Arnal, J.C. Juillien, S. Jallade, Recherches sur la transition aux vitesses elevees. Rapport de synthese final, ONERA-CERT (Toulouse), 27/5005.15 DY, 1989.

- [48] R.L. O'Neal, A.C. Bond, Heat transfer to 0° and 75° swept blunt leading edges in free flight at Mach numbers from 1.90 to 3.07, NASA report, TN D-1256, 1962.
- [49] W.J. Fleming, W.E. Krauss, Aerodynamic heating from turbulent boundary layer swept surfaces, in: Proceedings of the 3rd International Heat Transfer Conference, Vol. H, 1966, pp. 102–112.
- [50] C.P. Goyne, R.J. Stalker, A. Paull, Skin-friction measurements in high-enthalpy hypersonic boundary layers, J. Fluid Mech. 485 (2003) 1–32.
- [51] E.R.G. Eckert, Engineering relations for friction and heat transfer to surfaces in high velocity flow, J. Aeronaut. Sci. 22 (8) (1955) 585–587.
- [52] L.S.G. Kovasznay, Turbulence in supersonic flow, J. Aeronaut. Sci. 20 (1953) 657–682.
- [53] J. Gavaglio, Reynolds analogies and experimental study of heat transfer in the supersonic boundary layer, Int. J. Heat Mass Transfer 30 (5) (1987) 911–926.
- [54] H.H. Fernholz, M.A. Finley, Critical commentary on mean flow data for two dimensional compressible turbulent boundary layers, AGARDograph 253 (1980) 47–48.
- [55] EJ. Hopkins, M.W. Rubesin, M. Inouye, E.R. Keener, G.C. Mateer, T.E. Polek, Summary and correlation of skin Friction and heat transfer data for a hypersonic turbulent boundary layer over simple shapes, NASA report, TN D-5089, 1968.